Math 114
Fall 2016
Midterm 3 – Practice
xx/xx/16
Time Limit: 50 Minutes

Name (Print): ____________________

Recitation section: ____________________

This exam contains 8 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.

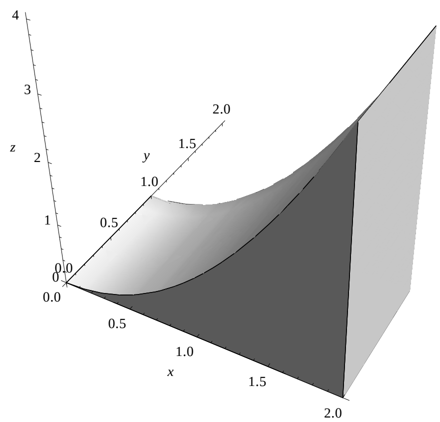
You are required to show your work on each problem unless stated otherwise.

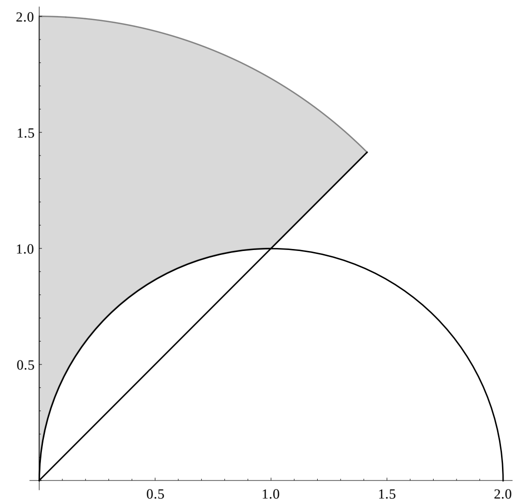
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- You must bring your PennID and have it out during the exam as someone could around to do an ID check.
- Once you finish the exam you must remain seated until the time has expired and your exam has been collected.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	15	
3	15	
4	15	
5	15	
Total:	70	

1. For these questions, you do not need to show your work — only provide an answer.





(a) (5 points) Which integrals compute the volume of the solid to the left? **Underline all correct:**

$$\int_0^1\int_0^1\int_0^1 dzdydx, \qquad \int_0^2\int_0^1\int_0^{x^2} dzdydx, \qquad \int_0^2\int_0^1 x^2dydx,$$

$$\int_0^4\int_0^{\sqrt{z}}\int_0^1 dydxdz, \qquad \int_0^2\int_0^2\int_0^{x^2} dzdxdy, \qquad \int_0^2\int_0^1 z^2dzdx.$$

Solution: Correct:

$$\int_0^2\int_0^1\int_0^{x^2} dzdydx, \qquad \int_0^2\int_0^1 x^2dydx, \qquad \int_0^4\int_0^{\sqrt{z}}\int_0^1 dydxdz$$

(b) (5 points) The region in the rightmost picture is described as $0 \le x \le y$, $x^2+y^2 \le 4$ and $(x-1)^2+y^2 \ge 1$. Which of the following alternatives describes the same region in polar coordinates?

Mark the correct alternative:

(I) $\pi/4 \le \theta \le \pi$ and $2/\cos(\theta) \le r \le 2$,
(II) $\pi/4 \le \theta \le \pi$ and $2/\sin(\theta) \le r \le 2$,
(III) $\pi/3 \le \theta \le \pi/2$ and $\sqrt{2}/2 \le r \le 2$,
(IV) $\pi/3 \le \theta \le \pi/2$ and $\cos(\theta) \le r \le 2$,
(V) $\pi/4 \le \theta \le \pi/2$ and $2\cos(\theta) \le r \le 2$,
(VI) $\pi/4 \le \theta \le \pi/2$ and $\cos(\theta) \le r \le 2$

Solution: The alternative $\pi/4 \leq \theta \leq \pi/2$ and $2\cos(\theta) \leq r \leq 2$ is the correct one.

2. (15 points) Integrate $f(x, y) = x^2 e^{-(x^2+y^2)^2}$ in the first quadrant, $x \geq 0$, $y \geq 0$.

$a)\ \pi/16 \qquad b)\ \pi/8 \qquad c)\ \pi/2 \qquad d)\ 1 \qquad e)\ \pi \qquad f)\ 2\pi$

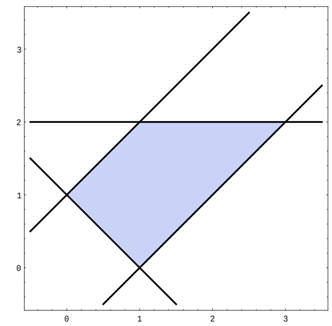
Solution: Switch to polar: $\pi/16$.

3. (15 points) Compute

$$\iint_D 3(x+y)dxdy$$

over the region D bounded by the lines $x-y=-1$, $x-y=1$, $x+y=1$ and $y=2$.

$a)$ 1 $b)$ 11 $c)$ 17 $d)$ 23 $e)$ 27 $f)$ 35



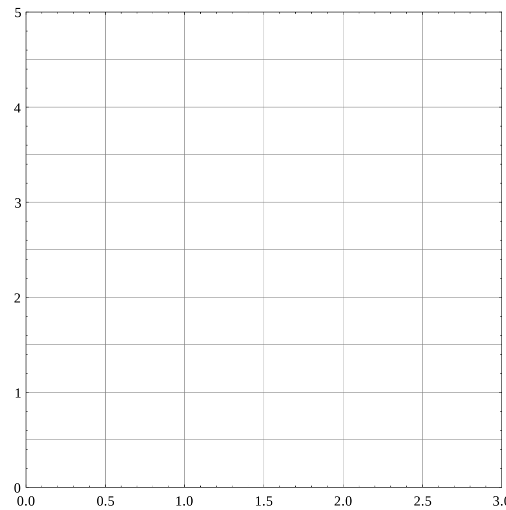
Solution: Do the change of coordinates $u=x-y$ and $v=x+y$. Answer is 23.

4. (15 points) Compute

$$\int_1^2 \int_{2/y}^2 y e^{\frac{1}{x}}\, dx dy + \int_2^4 \int_1^{4/y} y e^{\frac{1}{x}}\, dx dy.$$

Hint: Draw the regions and change order of integration.

a) $e^2 - e$ *b*) $3e^2 - 3e$ *c*) $6e - 6\sqrt{e}$ *d*) $7e^2 + 7e$ *e*) $4e^3 - 4e$ *f*) $3e^3 - 3e^2$



Solution: Answer: The region becomes $1 \leq x \leq 2$, $2/x \leq y \leq 4/x$ and the answer is $6e - 6\sqrt{e}$.

5. (15 points) The region E in the first octant[1] is defined as

$$E = \{(x, y, z) : xy \leq z \leq 2xy, \qquad y \leq x \leq 3y, \qquad x \leq z \leq 4x\}.$$

Rewrite the integral

$$\iiint_E \frac{z^2}{x^3y^3} dxdydz \qquad \text{in the form} \qquad \int_{a_1}^{a_2} \int_{b_1}^{b_2} \int_{c_1}^{c_2} f(u, v, w) dudvdw$$

by using the new coordinates $u = \frac{z}{xy}$, $v = \frac{x}{y}$ and $w = \frac{z}{x}$.

Solution: The Jacobian is $z/(x^2y^3)$, and the new integral is

$$\int_1^4 \int_1^3 \int_1^2 \frac{z^2}{x^3y^3} \cdot \frac{x^2y^3}{z} dudvdw = \int_1^4 \int_1^3 \int_1^2 \frac{z}{x} dudvdw = \int_1^4 \int_1^3 \int_1^2 w \cdot dudvdw.$$

Alternatively, solving for x, y, z we get

$$x = \frac{vw}{u}, \qquad y = \frac{w}{u}, \qquad z = \frac{vw^2}{u}$$

and one can go from there.

[1] Where $x, y, z \geq 0$

Extra page