

Math 114
Fall 2016
Midterm 1a
21/09/16
Time Limit: 50 Minutes

Name (Print): _____

Recitation section: _____

This exam contains 5 pages (including this cover page) and 7 problems. Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.

You are required to show your work on each problem on this exam.

- If you need more space, use the back of the pages; clearly indicate when you have done this.
- You must bring your PennID and have it out during the exam as someone could around to do an ID check.
- Once you finish the exam you must remain seated until the time has expired and your exam has been collected.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

1. (10 points) A plane given by $x + 3y - 2z = 2$ intersects a line parametrized by $(2, 0, 0) + (-1, 0, 1)t$. Find the x -coordinate of the intersection point.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Solution: Plug in the line parametrization into the equation of the plane: $(2 - t) + 3(0) - 2(t) = 2$. Solving this for t gives $t = 0$. We plug $t = 0$ into the line parametrization and get the point $(2, 0, 0)$. The x -coordinate is 2.

2. (10 points) Find a value of k such that if $\mathbf{u} = (3, 0, 1)$, $\mathbf{v} = (0, 2, 1)$ and $\mathbf{w} = (k, 2, 2)$, then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = 0$.

a) -2 b) 2 c) -3 d) 3 e) -4 f) 4

Solution: We compute the triple scalar product using the determinant formula:

$$\begin{vmatrix} 3 & 0 & 1 \\ 0 & 2 & 1 \\ k & 2 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ k & 2 \end{vmatrix} + 3(4 - 2) - 2k = 6 - 2k$$

This is 0 if $k = 3$.

3. (10 points) A plane has the equation $2x + y - z = 2$. Find the distance from $(5, 0, 0)$ to the plane.

a) $8/\sqrt{6}$ b) $4\sqrt{2}$ c) $4/3$ d) $2/3$ e) $4\sqrt{2}/\sqrt{3}$ f) $4/\sqrt{6}$

Solution: We begin with finding a point in the plane: $P = (1, 0, 0)$. Furthermore, the normal of the plane is $\mathbf{n} = (2, 1, -1)$. Let $S = (5, 0, 0)$. The distance formula gives the distance

$$\frac{|PS \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{(4, 0, 0) \cdot (2, 1, -1)}{\sqrt{4 + 1 + 1}} = 8/\sqrt{6}.$$

4. (10 points) The equations

1) $x^2 + 4y^2 = 6$ 2) $4x^2 + 4z^2 = y$ 3) $3x^2 + 5z^2 + 3y^2 = 6$

defines conic surfaces in space. One is an ellipsoid (E), one is a paraboloid (P) and one is a cylinder (C). Find the correct matching.

a) 1C-2E-3P b) 1E-2C-3P c) 1P-2C-3E
 d) 1C-2P-3E e) 1E-2P-3C f) 1P-2E-3C

Solution: We have, in order, a cylinder, a paraboloid and an ellipsoid: 1C-2P-3E.

5. (10 points) A particles position is given by the curve $\mathbf{r}(t) = (2t, t^2 - 1, 2 + 2t)$. At a certain point P on the curve, the tangent vector is parallel with the plane $2x + 3y - z = 2$. What is the x -coordinate of P ?

a) $-1/3$ b) $1/3$ c) $-2/3$ d) $2/3$ e) -1 f) 1

Solution: To be parallel with a plane is the same as being perpendicular to the normal of the plane. The tangent line direction is given by $\mathbf{r}'(t) = (2, 2t, 2)$. Thus, we seek t such that $\mathbf{r}'(t) \cdot (2, 3, -1) = 0$. We get the equation $4 + 6t - 2 = 0$, which is solved by $t = -1/3$.

The x -coordinate of $\mathbf{r}(-1/3)$ is $-2/3$.

6. (10 points) A golf ball is hit in an upwards angle of 60° and lands 5 seconds later. How fast across the ground did the ball travel in meter per second?

Use $g = 10$ meter per second².

a) $5\sqrt{3}$ b) $10\sqrt{3}$ c) 25 d) $25\sqrt{3}$ e) 50 f) $50\sqrt{3}$

Solution: The initial velocity of the ball is given by $s(\cos 60^\circ, \sin 60^\circ) = s(1/2, \sqrt{3}/2)$, where s is the initial speed. Integrating and adding the gravity contribution gives the position

$$\mathbf{r}(t) = s(1/2, \sqrt{3}/2)t - (0, 5)t^2.$$

After 5 seconds, we know that the y -coordinate is 0. Plugging in $t = 5$ in the y -component gives

$$5\sqrt{3}s/2 - 5 \cdot 5^2 = 0$$

so $s = 50/\sqrt{3}$. This is the initial *speed*, but we seek the x -component. This is given by $(50/\sqrt{3}) \cos 60^\circ = 25/\sqrt{3}$.

7. (10 points) A curve is parametrized as $\mathbf{v}(t) = e^t(\cos 2t, \sin 2t, 1)$, for $0 \leq t \leq 1$. Find the length of the curve.

a) $2(e - 1)$ b) $\sqrt{5}(e - 1)$ c) $\sqrt{6}(e - 1)$
d) $3(e - 1)$ e) $\sqrt{10}(e - 1)$ f) $\sqrt{11}(e - 1)$

Solution: We compute

$$\mathbf{v}'(t) = (e^t \cos(2t) - 2e^t \sin(2t), e^t \sin(2t) + 2e^t \cos(2t), e^t).$$

Then, $|\mathbf{v}'(t)|$ is given by

$$\begin{aligned} e^t \sqrt{(\cos(2t) - 2\sin(2t))^2 + (\sin(2t) + 2\cos(2t))^2 + 1^2} &= \\ e^t \sqrt{5(\cos^2(2t) + \sin^2(2t)) + 1} &= \\ e^t \sqrt{6} & \end{aligned}$$

The arc-length is then given by

$$\int_0^1 |\mathbf{v}'(t)| dt = \int_0^1 e^t \sqrt{6} dt = \sqrt{6}(e^1 - e^0) = \sqrt{6}(e - 1).$$